

### Laboratory Plasma Astrophysics Theory and Selected Topics

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### A little bit about me...





Young me setting up a pulsed-power conical jet experiment in 2014

### A little bit about me...

- Started as an astronomy student
- In January 2013 I joined the plasma physics lab at my university in Chile
- Fell in love with experimental plasma physics
- In 2018, I moved to London to do my PhD at Imperial College
- In 2022, I moved to Princeton Astrophysics as a postdoc
- Now I'm off to Livermore to kickstart my own research program, combining theory and experiments







### The universe is magnetic

### The Whirlpool galaxy and its magnetic fields

Stellar flares





*Plasma* phenomena are everywhere in the cosmos!

Credit: NASA, SOFIA, ESA, STScl v.valenzuela@princeton.edu

Accretion discs

Outline



- **1. Plasma Astrophysics**: Why is it important? Why is it difficult?
- 2. Laboratory Plasma Astrophysics: What is it? Where do we do it?
- 3. Theory of MHD scaling: From the cosmos to the lab
  - a. When can a plasma be treated as a fluid?
  - b. Dimensionless parameters and their meaning

### 4. Selected topics

- 1. Plasma jets and radiative/MHD instabilities
- 2. Turbulence and magnetic dynamo
- 3. Magnetic reconnection and plasmoids
- 4. Subcritical MHD shocks and collisionless magnetized shocks

# Laboratory Astrophysics with high-energy density (HED) facilities



- 1. Laboratory data providing important input parameters for astrophysical modelling
  - a. Spectral lines, excitation rates, recombination rates, etc. "traditional" laboratory astrophysics
  - b. Properties of matter in extreme conditions (temperature, density, pressure, etc) e.g opacity, equation of state, warm dense matter
- 2. Dynamical laboratory experiments:
  - a. Benchmarking hydrodynamics (compressible), radiation-hydrodynamics, magneto-hydrodynamics codes under scalable conditions
  - b. Laboratory experiments may prompt the emergence of new concepts

### Plasma astrophysics deals with fluids, fields and particles and how they behave in the universe





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#### Motivation: plasma astrophysics

# Fundamental processes in astrophysics are mediated by plasma effects







#### Cosmic ray acceleration in supernovae



Plasma physics is crucial for understanding

- Formation and evolution of structures
- **Electromagnetic sources sources** (e.g. stars, interstellar and -cluster medium, magnetospheres, compact objects)
- Astro-particles (e.g. cosmic rays)

### Plasmas often are highly dynamical, nonlinear, and multiscale: hard to interpret observations and model!



Observation of Sgr A\* in the Milky Way



*Global* simulation of a turbulent black hole accretion disc

Local simulation of a turbulent black hole accretion disc



Credit: Ripperda et al. (BHAC)

EHT Collaboration; Astrophys. J. (2024); Bambic et al., MNRAS (2024)

# Laboratory Plasma Astrophysics: take a subset of an astrophysical problem and study it in detail





#### **Experimental realization**

Reference (with hyperlink)

# I work with *dynamical* laboratory astrophysics experiments on high-energy density facilities



### High-energy density plasma (HEDP) facilities

Working definition: energy density  $\varepsilon \ge 10^{12}$  erg cm<sup>-3</sup>; pressure  $p \ge 1$  Mbar

#### Lasers

- ➤ Energy: ~  $1 10^4$  J → MJ
- Timescales of ps to 10s of ns

Laser facilities: Vulcan (UK), LIL (France), FIREX (Japan),

Plasma volumes ~ mm<sup>3</sup>

Omega (USA), NIF (USA), LMJ (France), etc

#### **Pulsed-power generators**

- ➢ Energy: ∼ 100 J to several MJ
- $\blacktriangleright$  Timescales of 100s of ns to  $\mu$ s
- Plasma volumes ~ cm<sup>3</sup>

Pulsed-power facilities: Z-machine (USA), MAGPIE (UK), COBRA (USA), PUFFIN (USA), etc









# Laboratory experiments are good for investigating fundamental physics of astrophysical objects in detail



- Study a subset of the problem
- Make simplifying assumptions (symmetry, disregarding some "unimportant" physics)
- Make educated guess on conditions / profiles

- Study a subset of the problem
- No simplifying assumptions (difficult to turn-off a physical package)
- Control of initial conditions

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# Laboratory experiments are good for investigating fundamental physics of astrophysical objects in detail



Scaled representation of astrophysical plasma dynamics:

 Laboratory and astrophysical phenomena are described by the same set of equations ideal MHD: *Rm* >>1; *R*e >>1

- Creating appropriate initial conditions (morphology)
- Dimensionless numbers and similarity transformations

### $(M_S, M_A, \beta)$

### Lab experiment reproducing the <u>evolution</u> of an astrophysical system

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# Hydrodynamic and magnetohydrodynamic flows often behave similarly despite their size





### A plasma can be treated as a fluid when the particles within are localized



#### Collisional



Particle-particle collisions are ٠ efficient

"Easy". Use Braginskii theory.

#### Semi-collisional



- Gyro-orbits are small ٠
- Particle-particle collisions are efficient ٠ along B-field

Harder. Anisotropic. Can use Braginskii theory but handle with care.

#### Collisionless



- Gyro-orbits are small
- Waves in the plasma efficiently pitchangle scatter the particles (effective collisionality)

#### Very hard. Possibly not universal. Handle case-by-case.

Braginskii, in: Review of Plasma Physics (1965); Marcowith et al., Rep. Prog. Phys. (2016); Kempski et al., Astrophys. J. (2025)

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Introduction: MHD scaling

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Introduction: MHD scaling

# Let us write the MHD equations with explicit dependence on scale



### Approach:

1. Write plasma and field quantities in terms of characteristic values and order-unity objects



2. Write differential operators using characteristic global spatial and temporal scales explicitly

$$\nabla = \frac{1}{L}\widehat{\nabla}; \qquad \frac{\partial}{\partial t} = \frac{1}{t_0}\frac{\partial}{\partial \hat{t}} \equiv \frac{u_0}{L}\frac{\partial}{\partial \hat{t}}$$

*L*: global spatial scale of the system  $t_0$ : hydrodynamic timescale  $u_0$ : characteristic flow velocity

- 3. Apply in all MHD equations
- 4. Solve such that left-hand-side of each equation such that it is dimensionless (only hat objects)

• The conservation of mass is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

• Introducing scaling variables

$$\rho = \rho_0 \hat{\rho}; \, \boldsymbol{u} = u_0 \hat{\boldsymbol{u}}; \qquad \text{Fluid parameters}$$
$$\nabla = \frac{1}{L} \widehat{\nabla}; \, \frac{\partial}{\partial t} = \frac{u_0}{L} \frac{\partial}{\partial \hat{t}} \qquad \text{Differential operators}$$

The continuity equation becomes

$$\frac{\rho_0 u_0}{L} \frac{\partial \hat{\rho}}{\partial \hat{t}} + \frac{\rho_0 u_0}{L} \widehat{\nabla} \cdot (\hat{\rho} \widehat{\boldsymbol{u}}) = 0 \qquad \rightarrow \frac{\partial \hat{\rho}}{\partial \hat{t}} + \widehat{\nabla} \cdot (\hat{\rho} \widehat{\boldsymbol{u}}) = 0$$

Continuity equation is always scale-invariant (no intrinsic  $L, t_0 = L/u_0$ )

#### Mass is conserved at all scales

Ryutov et al., Astrophys. J. (1999); Cross et al., Astrophys. J. (2016)

٠



### MHD scaling: momentum equation

• The conservation of momentum is

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} = -\boldsymbol{\nabla}p + \frac{1}{\mu_0}\boldsymbol{\nabla} \times \boldsymbol{B} \times \boldsymbol{B} + \rho \boldsymbol{\nu} \boldsymbol{\nabla}^2 \boldsymbol{u}$$

Introducing scaling variables

$$= \rho_0 \hat{\rho}; \boldsymbol{u} = u_0 \hat{\boldsymbol{u}}; p = p_0 \hat{p}; \boldsymbol{B} = B_0 \hat{\boldsymbol{B}}$$
$$\nabla = \frac{1}{L} \hat{\nabla}; \ \frac{\partial}{\partial t} = \frac{u_0}{L} \frac{\partial}{\partial \hat{t}}$$

Fluid + field parameters

Differential operators

• Solving for dimensionless left-hand-side (good homework)

ρ

$$\hat{\rho} \frac{\partial \hat{u}}{\partial \hat{t}} + \hat{\rho} (\hat{u} \cdot \hat{\nabla}) \hat{u} = -\frac{p_0}{\rho_0} \frac{1}{u_0^2} \hat{\nabla} \hat{p} + \left(\frac{B_0^2}{\mu_0 \rho_0}\right) \frac{1}{u_0^2} \hat{\nabla} \times \hat{B} \times \hat{B} + \frac{\nu}{u_0 L} \hat{\rho} \hat{\nabla}^2 \hat{u}$$
Sound speed:  $c_s^2 = p_0 / \rho_0$  Alfven velocity:  $V_A = B_0 / \sqrt{\mu_0 \rho_0}$  Viscous term is the only one with spatial scales  
Coefficient yields Sonic Mach number:  $M_s = u_0 / c_s$  Alfvenic Mach number:  $M_A = u_0 / V_A$  Coefficient yields the Reynolds number:  $Re = u_0 L / \nu$ 

In the limit  $Re \gg 1$ , momentum is conserved on (global) scales L, and the momentum equation is scale-invariant



### MHD scaling: induction equation



• The evolution of the magnetic field is

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \boldsymbol{u} \times \boldsymbol{B} + \eta \nabla^2 \boldsymbol{B}$$

• The dimensionless induction equation becomes (again, homework)

$$\frac{\partial \hat{B}}{\partial \hat{t}} = \hat{\nabla} \times \hat{u} \times \hat{B} + \frac{\eta}{u_0 L} \hat{\nabla}^2 \hat{B}$$

Resistive term is the only one with spatial scales

Magnetic Reynolds number:  $Rm = u_0 L/\eta$ 

Resistive dissipation breaks the frozen-in condition of ideal MHD

### In the limit $Rm \gg 1$ , the magnetic field is coupled to the fluid on scales *L*, and the induction equation is scale-invariant



• Adiabatic energy conservation

$$\frac{\partial p}{\partial t} + (\boldsymbol{u} \cdot \nabla)p = -\gamma p(\nabla \cdot \boldsymbol{u}) + \chi \nabla^2 p$$

Scales are coupled only to the **Peclet number:**  $Pe = u_0 L/\chi$ 

In the limit  $Pe \gg 1$ , heat is advected along the flow, thermal diffusion is negligible on scales *L*, and the energy equation is scale-invariant

# MHD scaling is based on comparing a small set of dimensionless parameters in the lab and astrophysics



Supersonic ( $M_s > 1$ ) fighter jet creating a shock in the atmosphere

### Related to the partition of internal and magnetic energy density, plasma- $\beta$ parameter

$$\beta \equiv p/p_M; \quad p_M \equiv B^2/2\mu_0; \quad \beta = 2 M_A^2/M_s^2$$



# Plasma flows are scale invariant when dissipation is negligible



Ryutov et al., Astrophys. J. (1999); Cross et al., Astrophys. J. (2016)

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# Dimensionless numbers in the universe, the laboratory, and numerical simulations





### Dimensionless parameters as ratios of length- and timescales



- A modern approach is to calculate **dissipation time- and length-scales**
- They are measurable in experiments (unlike dimensionless parameters), so they are practical/useful

*Ex:* take the Reynolds number  $Re = \frac{u_0 L}{v} = \frac{u_0 / L}{v / L^2} = \frac{1 / t_0}{1 / \tau_v} = \frac{\tau_v}{t_0}$  *Viscous timescale Re*  $\gg 1 \leftrightarrow \tau_v \gg t_0$ 

*Conclusion:* When Re is large compared to unity, time for viscous damping is much greater than hydrodynamic time. Viscosity is *inefficient* at dissipating *large-scale* momentum.

• Associated with this timescale there is a viscous length  $\ell_{\nu}$  at which large-scale momentum is dissipated

$$\tau_{\nu}(L = \ell_{\nu}) \sim t_{0} \quad \leftrightarrow \quad \frac{\ell_{\nu}^{2}}{\nu} \sim \frac{L}{u_{0}} \quad \leftrightarrow \quad \ell_{\nu}^{2} = \frac{\nu}{u_{0}L}L^{2} \quad \longrightarrow \quad \ell_{\nu} = Re^{-1/2}L$$

 $Re \gg 1 \rightarrow \ell_{\nu} \ll L$ : viscosity operates on small scales

Homework: what are the equivalent interpretations for Rm and Pe?

# In summary: what is the goal of dynamical laboratory astrophysics? (and what isn't)



- Dynamical laboratory astrophysics: study of evolution of astrophysics-relevant plasma flows from well-known initial conditions
- Simplify a complicated astrophysics system, then make sure scaling is appropriate (MHD, collisionless, Braginskii, etc)
- Make sure you measure all useful parameters: what is their structure? How do they evolve? This is what is interesting
- The universe is *very* large!
  - Even if you realize after the experiment that you targeted the wrong system initially, there probably is another system you can compare to
  - Collaboration with astronomers/astrophysicists helps
- Laboratory astrophysics is *not:* 
  - Recreation of full astrophysical systems
  - > A loose morphological comparison between lab and astro ("this looks like that")

### **Selected Topics**

(the good stuff)



### **Plasma jets in the universe**



# Plasma jets are full of substructures but remain collimated on large scales



#### They 'look' similar...

- Similar jet morphology
- Jets originate from a central source
- Spatial features along jet (shocks, knots)
- Terminal bow shocks





#### Protostellar jets

- Few light-years across
- Accretion into a protostar
- Speed ~ 100's km/s
- Optical emission (atomic lines)

#### Extragalactic jets

- Thousands light-years across
- Accretion into a supermassive black hole
- Speed ~ light
- Radio emission (synchrotron)

# Plasma jets are multi-scale where different regions should be treated with a different approximation





Frank et al., in: Protostars and Planets IV (2014); arXiv:14023553

# Far from the source, in the hydrodynamical stage, jets can be collimated by conical shocks





Cooling parameter: ratio of radiative cooling time to hydrodynamic time  $\chi \equiv \frac{\tau_{cool}}{R_j/V}$ 

31

### Pulsed-power experiments have shown the effect of radiative cooling in the structure of jets





#### Radiative cooling affects jet collimation

Stronger cooling

- Converging plasma flow, re-directed by a standing conical shock
- Radiatively cooled jet, M >10,  $\chi$  <1
- Jet velocity ~ 200km/s
  - Electron density 10<sup>18</sup>-10<sup>19</sup> cm<sup>-3</sup>
  - Pe > 10-50  $\lambda/R < 10^{-4}$  $Re > 10^4$

Lebedev et al., ApJ 2002

# Magnetically-driven jets can be created by using foils in pulsed-power experiments





Lebedev et al, MNRAS (2005), Ciardi et al, ApJL (2009), Suzuki-Vidal et al, PoP (2010, 2012)

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Introduction: MHD jets

33

# Well-known initial conditions in the experiments allow comparing with astrophysics codes



Simulations with laboratory code (GORGON) Ciardi et al, PoP 2007 Astrophysical code (ASTROBEAR) Huarte-Espinosa et al, 2012



Magnetic cavity ( $\beta <<1$ ) with dense jet on axis ( $\beta \sim 1$ ) MHD instabilities, change of magnetic field topology ( $B_{\phi} \rightarrow B_z$ ,  $B_r$ ) Energy balance: Poynting flux, magnetic energy, kinetic energy, radiation.

# MHD instabilities can develop gradually over the evolution of the jet



H. Calamy et al, 2007



do not destroy collimation

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268ns

а

b

### Descriptive title

278ns

Experiment versus 3-D MHD Ciardi et al, PoP 2007

298ns

288ns



4 mm

Jet driven by the pressure of the toroidal magnetic field

Collimation of the central jet by the hoop stress

Collimation of the magnetic bubble by the ambient medium

Two temporal scales for outflow variability:

- fast instability growth time (~1ns)
- slow bubble growth time (~50ns)

Instabilities do not destroy the jet but produce a clumpy outflow n<sub>i</sub> ~ 10<sup>19</sup> cm<sup>-3</sup>, T ~ 200 eV, I ~ 1 MA, B ~ 100 T Re > 10<sup>4</sup>,  $\lambda/R$  ~10<sup>-5</sup>, Pe > 10 β ~ 1, Re<sub>M</sub> ~ 50-300





### **Turbulent flows in a neutral fluid (liquid or gas)**



Van Dyke, *An Album of Fluid Motion* (1988) v.valenzuela@princeton.edu

# In a plasma, magnetic fields are entrained with the turbulence, producing much more complicated systems





Dynamo: Magnetic fields can be amplified by line stretching (shear, compression, rotation)

Beattie et al., Phys. Rev. Lett. (2025, in review)

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Introduction: MHD turbulence

# The TDYNO experiments create magnetized turbulence using lasers



Tzeferacos et al., Nat. Comm. (2018); White et al., Nat. Comm. (2019); Bott et al., PNAS (2021)

Results: MHD turbulence

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# First demonstration of fluctuation turbulent dynamo in the laboratory







#### B-field reconstruction from proton probing



### Magnetic reconnection is a common process in the universe that energizes the fluid from the field







### The Sweet-Parker model of magnetic reconnection

Conservation of mass: Reconnection rate:  $\frac{\# \text{ of lines out}}{\# \text{ of lines in}} = \frac{V_{\text{out}}}{V_{\text{in}}}$  $\rho V_{in}L = \rho V_{out}\delta \longrightarrow \frac{L}{\delta} = \frac{V_A}{V_{in}}$ Conservation of energy:  $\frac{B^2}{2\mu_0} = \frac{1}{2}\rho V_{out}^2 \rightarrow V_{out} = \frac{B}{\sqrt{\mu_0\rho}} \equiv V_A$ 2L(max outwards speed) Ohm's law  $\mathbf{E} + \mathbf{u} \times \mathbf{B} = \eta \mathbf{J} \rightarrow V_{in}B = \eta J_z$ Ampere's law:  $J_z \sim B/\mu_0 \delta$ 

 $\rightarrow \frac{V_{out}}{V_{in}} = \left(\frac{\mu_0 L V_A}{\eta}\right)^{1/2} \equiv S^{1/2}$ 

1. S is called **Lundquist number**, it is dimensionless (universal), and **sets the reconnection rate** 

2. SP magnetic reconnection **dissipates energy through Spitzer resistivity** 

## Magnetic reconnection can be chaotic, bursty, and explosive when plasmoids are formed





### Current sheet length L is not a free parameter in Sweet-Parker reconnection

### Plasmoid reconnection has been observed in pulsedpower experiments



Plasmoid unstable layer in Stable SP with aluminium plasma carbon plasma experiments [Hare+ 2017] [Suttle+ 2016] Pulsed-power magnetic reconnection experiments field of view **END VIEW** (a) 16mm (y) wire [Suttle+ 2016] reconnection layer



S = 20

44

S = 120



### Magnetic reconnection in laser-driven experiments





### **Plasmoid reconnection in laser-driven experiments**



Pearcy et al., Phys. Rev. Lett (2024)

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#### Results: plasmoids in laser-driven reconnection

### Magnetized shocks in the universe





Observations of the transition from the heliosphere have shown a continuous shock around the solar system





Burlanga et al., Astrophys. J. Lett. 2013

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#### Introduction: subcritical shock waves

# Magnetic fields and dissipation can drastically change the shape of a shock





Slide credit: Danny Russell (Imperial)

# Magnetic fields and dissipation can drastically change the shape of a shock





# Magnetic fields and dissipation can drastically change the shape of a shock



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# To create subcritical shocks, an exploder wire array blew magnetized plasma winds onto two cylindrical obstacles



(b

A supersonic, super-Alfvénic Plasma Flow with a frozen in B - Field is accelerated by the  $J \times B$  force



Russell et al., Phys. Rev. Lett. (2022); Journal Plasma Phys. (2023)

# The results show the formation of subcritical shocks where dissipation is given by resistivity





TABLE I. Characteristic plasma parameters upstream of the subcritical shock ~400 ns after current start (details in the Supplemental Material [34]). To evaluate the magnetic Reynolds number, a scale length of 10 mm, the distance between the wire array and the obstacles, gives  $\text{Re}_M \sim 10$  while a distance of ~0.8 mm, the subcritical shock width, gives  $\text{Re}_M \sim 1$ .

Dimensionless parameter		Value
Thermal beta	$\beta_{\rm th}$	1.7
Dynamic beta	$\beta_{\rm ram}$	18
Sonic Mach number	$M_S$	2.5
Alfvénic Mach number	$M_A$	3
Magnetosonic Mach number	$M_{\rm MS}$	1.9
Reynolds number	Re	$4 \times 10^4$
Magnetic Reynolds number	$\operatorname{Re}_M$	$10 \rightarrow 1$

#### Shock width ~ resistive scale Transition always happens at Rm = 1!

Russell et al., Phys. Rev. Lett. (2022); Journal Plasma Phys. (2023)

### Often shocks in astrophysics are collisionless



#### Supernova Remnant SN 1006



#### **Common features of collisionless shocks**

1. Hierarchy of length-scales



- 2. Interactions are mediated by collective EM fields
- 3. Efficient ultra high-energy particle accelerators

# Experiments at OMEGA and the NIF have created collisionless shocks mediated by the Weibel instability



Park et al., Phys. Plasmas (2015); Huntington et al., Nature Phys. (2015)

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## Experiments at the NIF have created fully-developed Weibel-mediated shocks



NITE

400



experiments and young SNR shocks			
Parameter	NIF experiments	Typical young SNR (for example, SN 1006)	
Shock velocity (km s <sup>-1</sup> )	1,000–2,000	3,000-5,000	
Ambient magnetic field (G)	2×10 <sup>4</sup>	3×10 <sup>-6</sup>	
Ambient plasma density (cm <sup>-3</sup> )	5×10 <sup>19</sup>	0.2	
Ambient plasma temperature (eV)	500	1	
System size (cm)	2.5	3×10 <sup>19</sup>	
Collisionality ( $L_{system}/L_{m.f.p.}$ )	0.03	0.01	
Sonic Mach number $(v_{sh}/c_s)$	12	400	

400

Fiuza et al., Nat. Phys (2020)

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#### Results: collisionless shocks

Alfvén Mach number  $(v_{sh}/v_{A})$ 

### Conclusions

- HEDP facilities can create macroscopic quantities of matter at astrophysical conditions
- Dynamical laboratory experiments:
  - a) benchmarking hydrodynamics (compressible), radiationhydrodynamics, magneto-hydrodynamics codes under scalable conditions.
  - b) laboratory experiments may prompt the emergence of new concepts

### The "Grand Challenge": ability <u>to measure</u> all required parameters (n, T, $V_{flow}$ , B, etc.), instead of inferring them from computer models



4 mm

 $B.L \ (\mu mMG)$ 



